NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

October Examination 2019

Course Code: **MTH 305**

Course Title: **Complex Analysis II**

Credit Unit: **3**

Time Allowed: **3 Hours**

Instruction: **Answer Question Number One and Any Other Four Questions**

1. (a) Perform each of the indicated operations:
   i. $(2+7i)(11-5i)$ **(2 marks)**
   ii. $(-1+2i)\{(7-5i)+(-3+4i)\}$ **(2 marks)**
   iii. $\frac{5+5i}{3-4i}+\frac{20}{4+3i}$ **(2 marks)**

   (b) Solve the following
   i. Find real numbers x and y such that $3x+2iy-ix+5y=7+5i$ **(4 marks)**
   ii. Evaluate $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ **(4 marks)**

   (c) Suppose $A(x,y)=2xy-ix^2y^3$. Find (a) grad A, (b) div A, (c) Laplacian of A. **(8 marks)**

2. (a) Solve the equation $z^2+(2i-3)z+5=0$. **(6 marks)**

   (b) Express each equation in terms of conjugate coordinates:
   i. $2x+y=5$, **(2 marks)**
   ii. $x^2+y^2=36$ **(2 marks)**
   iii. Determine the image of the point $P, z=3+i2$, on the $w-plane$ under the transformation $w=3z+2-i$. **(2 marks)**

3. (a) Show that the real and imaginary parts of the function defined by $f(z)=z^2$ are harmonic.

   (4 marks)

(b)Show that $u(x,y) = x^3y - y^3x$ is an harmonic function and find the function $v(x,y)$ that is conjugate to $u(x,y)$. **(8 marks)**

4. (a) Evaluate the integral $\int_C f(z)\,dz$ where $f(z) = (z - i)^2$ and C is the straight line joining $A\ (z = 0)$ to $B\ (z = 1 + i2)$. **(6 marks)**
(b) Prove that: (i) $\frac{d}{dz}\sin^{-1} z = \frac{1}{\sqrt{1-z^2}}$, (ii) $\frac{d}{dz}\tan^{-1} z = \frac{1}{1-z^2}$ **(6 marks)**

5. Verify Cauchy's theorem by evaluating the integral $\oint_C f(z)\,dz$ where $f(z) = z^2$ around the square formed by joining the points $z = 1, z = 2, z = 2 + i, z = 1 + i$. **(12 marks)**

6. Find the residues at all the poles of $f(z) = \frac{3z}{(Z+2)^2(Z^2-1)}$.
$f(z)$ has a pole of order 2 (a double pole) at $z = -2$ and two poles of order 1 (simple poles) at $z = \pm 1$. **(12 marks)**