NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code: MTH312
Course Title: Abstract Algebra
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

1a) Define the following terms: i) $\mathrm{Im}\ f$, f is a homomorphism. ii) $\mathrm{Ker}\ f$, f is a homomorphism. iii) Commutative ring. iv) an Alternating group. **(8 marks)**

b) Show that if $f: G_1 \longrightarrow G_2$ is a group homomorphism. Then f is injective if and only if $\mathrm{Ker}\ f = \{e_1\}$. Where e_1 is the identity element of the group G_1.**(7 marks)**

c) Define a Sylow p-subgroup ii)State without prove the first Sylow's theorem.**(7 marks)**

2a) Define i). a ring homomorphism ii). An epimorphism**(4 marks)**

bi) Let R be a ring. Show that the identity map I_R is a ring homomorphism. What are $\mathrm{Ker}\ I_R$ and $\mathrm{Im}\ I_R$? Is I_R an epimorphism?

bii) Let $s \in \mathbb{N}$, show that the map $f: \mathbb{Z} \longrightarrow \mathbb{Z}_s$ given by $f(m) = m$ for all $m \in \mathbb{Z}$ is a ring homomorphism. What are $\mathrm{Ker}\ f$ and $\mathrm{Im}\ f$? Is f an epimorphism? **(8 marks)**

3a) Define the terms i).Principal ideal ii). Nilpotent iii). Nil radical of R.**(6 marks)**

b) Given a ring R and an ideal I. Show that R/I is a ring with respect to addition and multiplication defined by $(x + I) + (y + I) = (x + y) + I$ and $(x + I)(y + I) = (xy) + I$ for all $x, y \in R$.**(6 marks)**

4a) Show that $\mathrm{Aut}\mathbb{Z} \cong \mathbb{Z}_2$ **(6 marks)**

b) Show that any cyclic group is isomorphic to $(\mathbb{Z}, +)$ or $(\mathbb{Z}_n, +)$. **(6 marks)**

5a) Define the terms

i) ideal of a ring. ii) proper ideal of a ring. iii) The ideal generated by $a_1, a_2, \cdots, a_n$, elements of a ring. **(6 marks)**

bi) Given that X is an infinite set and I is the class of all finite subsets of X. Show that I is an ideal of $\wp(X)$.

bii) For any ring R and $a_1, a_2 \in R$. Show that $Ra_1 + Ra_2 = \{x_1a_1 + x_2a_2 \in R\}$ is an ideal of R.

(**6 marks**)

6a). Explain the following terms i.) when a permutation is called r-cyclic. ii) A transposition. iii). When two cycles are said to be disjoint. iv) The signature of $f \in S_n$.**(6 marks)**

b) Express each of the following permutations as products of disjoint cycles.

i. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 2 & 1 & 3 \end{pmatrix}$ ii. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 3 & 1 & 2 \end{pmatrix}$

iii. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 4 & 7 & 2 & 1 & 3 & 6 & 5 \end{pmatrix}$

(3 marks)

c) Given that $f, g \in S_n$, show that $\mathrm{sign}(f^\circ g) = (\mathrm{sign}\ f)(\mathrm{sign}\ g)$.**(3 marks)**