



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi, Abuja**  
**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**2022\_2 Examinations**

**Course Code: MTH312**

**Course Title: Abstract Algebra**

**Credit Unit: 3**

**Time Allowed: 3 Hours**

**Total: 70 Marks**

**Instruction: Answer Question One (1) and Any Other 3 Questions**

1a) Define the following terms: i)  $\text{Im } f$ ,  $f$  is a homomorphism. ii)  $\text{Ker } f$ ,  $f$  is a homomorphism. iii) Commutative ring. iv) an Alternating group. **(8 marks)**

b) Show that if  $f: G_1 \rightarrow G_2$  is a group homomorphism. Then  $f$  is injective if and only if  $\text{Ker } f = \{e_1\}$ . Where  $e_1$  is the identity element of the group  $G_1$ . **(8 marks)**

c) Define a Sylow  $p$ -subgroup ii) State without prove the first Sylow's theorem. **(9 marks)**

2a) Define i). a ring homomorphism ii). an epimorphism **(6 marks)**

bi) Let  $R$  be a ring. Show that the identity map  $I_R$  is a ring homomorphism. What are  $\text{Ker } I_R$  and  $\text{Im } I_R$ ? Is  $I_R$  an epimorphism?

bii) Let  $s \in \mathbb{N}$ , show that the map  $f: \mathbb{Z} \rightarrow \mathbb{Z}_s$  given by  $f(m) = m$  for all  $m \in \mathbb{Z}$  is a ring homomorphism. What are  $\text{Ker } f$  and  $\text{Im } f$ ? Is  $f$  an epimorphism? **(9 marks)**

3a) Define the terms

i). ideal of a ring ii).proper ideal of a ring iii). The ideal generated by  $a_1, a_2, \dots, a_n$ , elements of a ring. **(7 marks)**

b)i. Given that  $X$  is an infinite set and  $I$  is the class of all finite subsets of  $X$ . Show that  $I$  is an ideal of  $\wp(X)$ .**(4 marks)**

ii.For any ring  $R$  and  $a_1, a_2 \in R$ . Show that  $Ra_1 + Ra_2 = \{x_1a_1 + x_2a_2 \in R\}$  is an ideal of  $R$ . **(4 marks)**

4a) Explain the following terms i. when a permutation is called r-cyclic. ii. a transposition iii. when two cycles are said to be disjoint iv. the signature of  $f \in S_n$ . **(7 Marks)**

b) Express each of the following permutations as products of disjoint cycles.

i.  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 2 & 1 & 3 \end{pmatrix}$     ii.  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 3 & 1 & 2 \end{pmatrix}$   
iii.  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 4 & 7 & 2 & 1 & 3 & 6 & 5 \end{pmatrix}$

**(4 Marks)**

c) Given that  $f, g \in S_n$ , show that  $\text{sign}(f \circ g) = (\text{sign } f)(\text{sign } g)$

**(4 Marks)**

5a) Show that  $\text{Aut}\mathbb{Z} \cong \mathbb{Z}_2$

**(7 Marks)**

b) Show that any cyclic group is isomorphic to  $(\mathbb{Z}, +)$  or  $(\mathbb{Z}_n, +)$ .

**(8 Marks)**

6a) Define the following terms

(i) Principal ideal

**(2 Marks)**

(ii) Nilpotent

**(2 Marks)**

(iii) Nil radical of  $R$ .

**(3 Marks)**

b) Given a ring  $R$  and an ideal  $I$ . Show that  $R/I$  is a ring with respect to addition and multiplication defined by  $(x + I) + (y + I) = (x + y) + I$  and  $(x + I)(y + I) = (xy) + I$  for all  $x, y \in R$ . **(8 Marks)**