

**NATIONAL OPEN UNIVERSITY OF NIGERIA**

**Plot 91, Cadastral Zone, NnamdiAzikiwe Expressway, Jabi, Abuja.**

**FACULTY OF SCIENCES**

**January\February Examination 2018**

**Course Code: MTH312**

**Course Title: Abstract Algebra II**

**Credit Unit: 3**

**Time Allowed: 3 HOURS**

**Instruction: ATTEMPTNUMBER ONE (1) AND ANY OTHERFOUR (4) QUESTIONS**

1. (a) Given that . Show that the composition  and  is a
2. homomorphism

(ii) What is Ker f(gof)?

(iii) FindIm(gof) **(2 Marks)**

(ii) Show that every subgroup H of an abelian group G is a normal subgroup. **(4 Marks)**

(b) Given H as the subgroup of S3 consisting of elements (1) and (12) and W consisting of permutations (1), (123), and (132), show that H is not a normal subgroup but W is a normal subgroup. **(8 Marks)**

(c) Given that N is a subgroup of G. Show that the following statement are equivalent

i). the subgroup N is normal in G **(2 Marks)**

ii). for all,  **(3 Marks)**

iii). For all  ,  **(3 Marks)**

1. (a) let  be a group homomorphism, show that
2. Ker f is normal subgroup of G1  **(4 Marks)**
3. Im f is a subgroup of G2 **(4 Marks)**

(b) If  is an onto group homomorphism and S is a subset that generates G1, show that f(S) generates G2  **(4 Marks)**

1. (a) If H and K are subgroups of a group G, with K normal in G, show that 

**(4 Marks)**

(b) If a group G be the internal direct product of its subgroups H and K, show that:

1. Each can be uniquely expressed as x=hk, where **(4 Marks)**
2. hk=kh  **(4 Marks)**

4 (a) Let R be a Boolean ring (i.e., show that also show that R must be commutative **(3 Marks)**

(b) Let R be a ring, if for a,b,c elements of R, show that

1. a0=0=0a **(3 Marks)**
2. a(-b)=(-a)b=-(ab) **(3 Marks)**
3. (-a)(-b)=ab **(3 Marks)**

5 (a) (i) Find the principal ideals of Z10 generated by and  **(3 Marks)**

(ii) Find the nil radicals of Z8 and p(X) **(3 Marks)**

(b) Let R be a ring with identity 1, if 1 is an ideal of R and, show that 1=R **(6 Marks)**

1. a) State and prove the Cayley’s theorem. **(6 Marks)**

b) Use the Cayley’stable to show that  is a group **(6 Marks)**