NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
November Examination 2018

Course Code: **MTH312**
Course Title: **Abstract Algebra II**
Credit Unit: **3**
Time Allowed: **3 HOURS**
Instruction: **ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Define a normal subgroup of a group G (3marks)

(b) Prove that if $H \trianglerighteq G,$ and $K \trianglerighteq G, K \leq G$. Then

(i) $HK \leq G$ (ii) $HK \trianglerighteq G$. (8marks)

(c) Define the term Ring (2marks)

(d) Let R be a ring. Show that the identity map 1_R is a ring homomorphism.

What are $\ker 1_R$ and $\text{im}1_R$.? (9marks)

2. (a) Let G be the group of integers under addition and let $\phi : G \rightarrow G$ such that $\emptyset(x) = 2x, \forall x \in G$.

Then prove that $\emptyset$ is a homomorphism (4marks)

(b) Show that the product of two normal subgroup is a normal subgroup (4marks)

(c) Let G be a group. Then show that $[G, G]$ is a normal subgroup of G. Hence show that $G/[G, G]$ is commutative group(4marks)

3. (a) Write the following permutations as the product of disjoint cycles

(i) $f = \begin{pmatrix} 123456789 \\ 234516798 \end{pmatrix}$ (ii) $f = \begin{pmatrix} 123456 \\ 654312 \end{pmatrix}$ (6marks)

(b) Find the inverse of each of the following permutations

(i) $\begin{pmatrix} 1234 \\ 1342 \end{pmatrix}$ (ii) $\begin{pmatrix} 1234 \\ 3412 \end{pmatrix}$ (6marks)

4. (a) Define ring homomorphism (2marks)

(b) Show that every homomorphic image of a commutative ring is commutative (5marks)

(c) Show that the quotient group of a cyclic group is cyclic (5marks)

5. (a) Let $f: R \to S$ be an onto ring homomorphism, show that if J is an ideal of S, then $f(f^{-1}(J)) = J$ (4marks)

(b) If H is any subgroup of a group G and $a, b \in G$. then prove that $Ha = Hb \Leftrightarrow ab^{-1} \in H$ (4marks)

(c) If $f: G \to H$ is an isomorphism of groups and G is an abelian group. Then show that H is also abelian (4marks)

6. (a) Define a sylow p-subgroup of a group G. (3marks)

(b) If H is a sylowp-subgroup of G and $x \in G$.

Then show that $x^{-1}Hx$ is also a sylow p-subgroup of G. (9marks)