NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code:	**MTH 312**
Course Title:	**Abstract Algebra II**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total Marks:	**70**
Instructions:	**Answer Question Number One and Any Other Four Questions**

1. (a) (i) Define a group and a subgroup. **(5 Marks)**
   (ii) Show that every subgroup of a Commutative group is normal. **(4 Marks)**

   (b) (i) Let H and K be normal subgroups of a group G, show that $H \cap K \lhd G$. **(3 Marks)**
   (ii) Show that the subgroup $< (1\ 2) >$ of S_3 is not normal. **(4 Marks)**
   (c) Show that every subgroup of a group G of index 2 is normal **(6 Marks)**

2. (a) (i) Define the term group homomorphism. **(2 Marks)**
   (ii) let $H \lhd G$, consider the map $\alpha: G \rightarrow G/H$ defined by $\alpha(x) = Hx$.
   Show that: (i) α is a homomorphism. **(3 Marks)**
   (ii) α is onto and find $Ker\ \alpha$ **(3 Marks)**
   (b) Show that $3\mathbb{Z}/12\mathbb{Z} \cong \mathbb{Z}_4$ **(4 Marks)**

3. (a) Define Sylow p-subgroup of a group G. **(3 Marks)**
   (b) (i) Define the internal product of two subgroups of a group **(3 Marks)**
   (ii) Hence, show that any group of order 15 is cyclic. **(6 Marks)**

4. (a) Let $R \neq \emptyset$ be a set and S a subset of R. Define (i) a ring R and (ii) a subring S as algebraic structures. **(4 Marks)**

   (b) (i) Show that $(\mathbb{Z}_n, +, \cdot)$ is a ring, where $\mathbb{Z}_n$ is the set of integers modulo n **(4 Marks)**

   (ii) Write out the Cayley tables for addition and multiplication in $\mathbb{Z}_6$. **(4 Marks)**

5. (a) (i) Define an Ideal of a ring R. **(2 Marks)**

(ii) Let R be a ring and I be an ideal in R, show that R/I is a ring with respect to addition and multiplication defined by: $(x + I) + (y + I) = (x + y) + I$ and $(x + I).(y + I) = xy + I \; \forall x, y \in R$. **(6 Marks)**

(b) Obtain two proper non-trivial subrings of $\mathbb{Z} \times \mathbb{R}$. **(4 Marks)**

6. (a) $f: R \rightarrow S$ be a ring homomorphism, where R and S are rings. Then show that $R/Ker\, f \cong Im\, f$. Hence, if f is surjective, then $R/Ker\, f \cong S$. **(9 Marks)**

(b) If $f: G \rightarrow H$ is an isomorphism of groups and G is abelian, then show that H is also abelian. **(3 Marks)**