NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code:	**MTH315**
Course Title:	**Analytical Dynamics**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

1. (a) Determine the number of degrees of freedom in each of the following cases:

   (i) 20 particles moving freely in a plane **(2 marks)**

   (ii) 15 particles moving freely in space . **(2 marks)**

   (b) A system of particles consists of a 3-gram mass located at (1, 2, -1), a 5-gram mass at (0,1,3) and 2-gram mass at (1, -1, 1). Find the center of mass. **(6 marks)**

   (c) A uniform beam is 10m long and has a mass 10kg and masses of 6kg and 8 kg are suspended from its ends; at what point must the beam be supported so that it may rest horizontally? **(8marks)**

   (d) A quadrilateral ABCD has masses 2,3, 5 and 7 units located at its vertices, A(3,-2,2), B(2,-2,3), C(1,-2,4) and D(4,1,3). Find the coordinates of the center of mass. **(4 marks)**

2. Three particles of masses 1,2,3 respectively have position vectors
   $r_1 = (t^2 + 4)i - t^2 j + tk,$
   $r_2 = -2ti + 3t^2 j - 2tk,$
   $r_3 = -t^2 i - t^2 j + 2tk$, where t is time.
   Find (a) the velocity of the center of mass at time $t = 0$ **(6 marks)**
   (b) the acceleration at t=1. **(6 marks)**

3. A particle of mass 12 units moves along a space curve whose position vector is given as a function of time t by $r = (t^4 - 3t)i + 6t^3 j + t^3 k$
   At time $t = 1$, find the (a) momentum **(6 marks)**
   (b) force field. **(6 marks)**

4. A particle of mass 3 moves in a force field depending on time t given by $F = 9t^2 i - 3tj + 6tk$. Assuming that at $t = 0$ the particle is located at $r_0 = -i - j + 2k$ and has velocity $v_0 = 3i + j - k$,
   find (a) the velocity **(6 marks)**
   (b) the position at any time t. **(6 marks)**

5. A particle moves along the x axis in a force field having potential $V = \frac{\alpha}{3}x^3 - \frac{\beta}{2}x^2$, where α and β are positive constants. Determine the point(s) of equilibrium. **(12 marks)**

6. (a) State without proof the Liouville's theorem in Hamiltonian theory. **(4 marks)**
   (b) Minimise the integral
   $I = \int_0^{\frac{\pi}{2}} \left[2\left(\frac{dy}{dt}\right)^2 - 2y^2 + 4ty\right] dt$, $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = 0$ **(8 marks)**