NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
November Examination 2018

Course Code:	**MTH315**
Course Title:	**Analytical Dynamics**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One and Any Other 4 Questions**

1. (a) State the Lagrange's equations **(2 marks)**
   (b) A system of two particles is connected by a string over a fixed, frictionless pulley.
   (i) Find the Lagrangian of the system and determine the equation of motion of system **(6 marks)**

   (c)The displacement of a particle from the origin is given by $\mathbf{r} = t^2\mathbf{i} + t^{-2}\mathbf{j}$, where is t the time. Find the velocity, speed and acceleration. At what time are the velocity and displacement right angles? **(7marks)**

   (d) A train moving with constant acceleration passes three posts, A, B, C on a straight road. The distance from A to B is 15m, and from B to C 20m. The train takes 6 sec to go from A to B and 5 sec to go from B to C. Find the acceleration of the train and its distance from A when its speed is 5.5 m/sec. **(7 marks)**

2. (a) Let m_1, m_2 be the masses of the two spheres moving in the same straight line with their respective velocities u_1, u_2 collide. If v_1, v_2 are their velocities after impact. Find their subsequent speeds if the coefficient of restitution is e. **(6 marks)**
   (b) Briefly explain the followings
   (i) a perfectly inelastic collision **(3 marks)**
   (ii) a perfectly inelastic collision. **(3 marks)**

3. (a) Three smooth spheres, A, B, C of masses m, 2m and 4m respectively rest on a smooth plane (horizontal) with their centres collinear, and B lies between A and C. The coefficients of restitution between any two pairs are equal. If A is projected towards B with velocity U and C moves with velocity. **(6 marks)**
   (b) A bullet of mass 1 g is fired into a block of ice of mass I kg which can slide freely. If the bullet is fired with speed 2000m/s and comes to rest embedded in the ice, determine the final speed of the ice. **(6 marks)**

4. (a) State the Hamilton's principle. **(2 marks)**

(b) If the Hamiltonian $H = \sum_{\alpha=1}^{n} P_\alpha \dot{q}_\alpha - L$ does not contain the variable time t explicitly and is expressed as a function of coordinate q_α and momemta p_α, prove that

$$\dot{p}_\alpha = -\frac{\partial H}{\partial q_\alpha}, \quad \dot{q}_\alpha = \frac{\partial H}{\partial p_\alpha},$$ **(5 marks)**

(c) A particle moves in the xy plane under the influence of a central force depending only on its distance from the origin. Determine the Hamiltonian for the system. **(5 marks)**

(5) (a) State Euler's equation **(2 marks)**

(b) Hence, find function y that minimize the integral $I = \int_0^{\frac{\pi}{2}} \left((\frac{dy}{dt})^2 - y^2 + 2ty \right) dt$

Subject to y(0) =0 and $y\left(\frac{\pi}{2}\right) = 0$. **(5 marks)**

(c) Prove that a transformation is canonical if there exists a function G

such that $\frac{dG}{dt} = L - l$where L and *l*are the Lagrangians of the old and new coordinates respectively and G is the generating function. **(5 marks)**

(6) (a) A body moves in a conservative force field
F= $(y^2\text{-}2xyz^3)\mathbf{i} + (3+2xy - x^2z^3)\mathbf{j} + (6z^3\text{-}3\,x^2z^2y)\,\mathbf{k}$ from the point (2,-1, 2) to (-1, 3, -2) Find the work done. **(6 marks)**

(b) If a force F is such that F = - ∇V where Vis the potential function and T is the kinetic energy, show that T +V = constant. **(6 marks)**