NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
2021_2 Examinations

Course Code: MTH 382
Course Title: Mathematical Methods IV
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

Q1. (a) Define each of the following:

(i) a partial differential equation **(2 marks)**
(ii) order of a partial differential equation **(2 marks)**

(b) Find

(i) $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$ **(2 marks)**

(ii) $\frac{\Gamma(6)}{2\Gamma(3)}$ **(3 marks)**

(c) Show that $J_{-n}(x) = (-1)^n J_n(x)$ for $n = 1,2,3,\ldots$ **(7 marks)**

(d) Solve the boundary value problem $\frac{\partial U}{\partial x} = 4\frac{\partial U}{\partial y}, U(0,y) = 8e^{-3y}$ by method of separation of variables. **(7 marks)**

Q2. (a) Define a Legendre function. **(2 marks)**

(b) State the Rodrigue's formula. **(2 marks)**

(c) Given that $P_0(x) = 1$ and $P_1(x) = x$. Find using the Recurrence formula

i) $P_2(x)$ **(3 marks)**

ii) $P_3(x)$ **(4 marks)**

Q3. (a) Define each of the following:

i) an ordinary differential equation **(2 marks)**
ii) a Legendre equation **(2 marks)**

(b) Evaluate $\int_0^1 t^4\,(1-t)^3 dt$ **(4 marks)**

(c) Establish that $\Gamma(\alpha+1) = \alpha!$ **(4 marks)**

Q4. (a) Define each of the following partial differential equation:

(i) $\frac{\partial U}{\partial t} = k\nabla^2 U$ **(1 mark)**

(ii) $\nabla^2 v = 0$ **(1 mark)**

b) Determine whether each of the following partial differential equations are linear or nonlinear. State the order of the equation and the name of the dependent and independent variables:

i) $\frac{\partial U}{\partial t} = 4\frac{\partial^2 U}{\partial x^2}$ **(1 mark)**

ii) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$ **(1 mark)**

iii) $\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 = 1$ **(1 mark)**

c) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ by method of separation of variables. **(7 marks)**

Q5 (a) State the boundedness and Lipschitz conditions. **(5 marks)**

(b) Find $P_n(x)$ by the Rodrigue's formula for $n = 0,1$ and 2. **(7 marks)**

Q6. (a) Define each of the following:

(i) Hypergeometric functions. **(2 marks)**
(ii) Bessel's equation of index v.**(2 marks)**

(b) Show that $2F(\alpha,\beta,\beta,x) = (1-x)^{-\alpha}$ **(4 marks)**

(c) Prove that $P'_{n+1}(x) = (2n+1)P_n(x) + P'_{n-1}(x)$, $n = 1,2,$ ...**(4 marks)**