NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_1 Examination

Course Code: MTH 382
Course Title: Mathematical Methods IV
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question One (1) and Any Other 4 Questions

Q1 (a) Define each of the following:

i) an ordinary differential equation **(2 marks)**
ii) a Legendre equation **(2 marks)**

(b) Show that if $R(p) > 0$ and $R(q) > 0$ then

$\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$ **(7 marks)**

c) Using the Rodrigue's formula, find $P_n(x)$ for $n = 0,1$ and 2. **(6 marks)**

d) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ by method of separation of variables. **(5 marks)**

Q2 (a) State the boundedness and Lipschitz conditions. **(5 marks)**

(b) Show that under the stated assumptions on F, the equation $F'(x) = F[x, f(x)], f(x_0) = y_0$ has a unique solution defined in the interval $(x_0 - r, x_0 + r)$

where $r < \min(a, \frac{b}{M}, \frac{1}{K})$ **(7 marks)**

Q3 (a) Define each of the following:

(i) a gamma function **(2 marks)**
(ii) a beta function **(2 marks)**

(b) Find (i) $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$ **(2 marks)**

(ii) $\Gamma(1) = 1$ **(3 marks)**

c) Prove that $(\alpha)_n = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}$ **(3 marks)**

Q4 (a) Define each of the following:

(i) Hypergeometric functions **(2 marks)**
(ii) Bessel's equation of index v **(2 marks)**

(b) Show that $2F(\alpha, \beta, \beta, x) = (1-x)^{-\alpha}$ **(3 marks)**

c) Find $\int_0^{\frac{\pi}{2}} J_0(zcos\theta)cos\theta d\theta$ **(5 marks)**

Q5 (a) Define a Legendre function? **(2 marks)**

(b)i) State the Rodrigue's formula **(2 marks)**

ii) Find $P_2(x)$ using the Rodrigue's formula **(4 marks)**

c) Prove that $P'_{n+1}(x) = (2n+1)P_n(x) + P'_{n-1}(x), n = 1,2, \ldots$ **(4 marks)**

Q6 (a) i) Define a partial differential equation **(1 mark)**

ii)What is the name of each of the following partial differential equation?

I. $\Delta^2\theta = \frac{1}{c}\frac{\partial^2\theta}{\partial t^2}$ **(1 mark)**
II. $\Delta^2\theta + f = 0$ **(1 mark)**
III. $\Delta^2\theta = 0$ **(1 mark)**

(b) Find the solution of the heat conduction equation $\alpha^2 u_{xx} = u_x$ with boundary condition $u_x(0.t) = 0, u_x(l,t) = 0$ **(8 marks)**