



**NATIONAL OPEN UNIVERSITY OF NIGERIA**  
**University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway, Jabi, Abuja.**

**FACULTY OF SCIENCES**  
**DEPARTMENT OF MATHEMATICS**  
**2022\_2 Examination**

**Course Code: MTH 382**  
**Course Title: Mathematical Methods IV**  
**Credit Unit: 3**  
**Time allowed: 3 HOURS**  
**Instruction: Answer Question One and any Other Three Questions**

1. (a) Define Legendry function (3 marks)  
(b) Prove that

$$P_n^1 + l(x) = xP_n^1(x) + (n + 1)P_n(x)$$

- (c) Define a Periodic Function  
(d) Consider the Laplace equation in polar co-ordinates  $u = \frac{1}{z}u_2 \frac{1}{2} + u_2 \frac{1}{z^2}u\theta\theta$  with boundary condition  $u(a, \theta) = f(\theta)$   $f$  is a given function on  $0 \leq \theta \leq 2\pi$  in order that  $u(z, 0)$  the single value, it is necessary that a function of  $\theta$ , show that  $u$  must be periodic with period  $2\pi$ .  
(2) Solve the Laplace equation

$$u_{xx} + u_{yy} = 0$$

In the rectangle  $0 < x < a, 0 < y < b$ , and which satisfies the boundary condition.

$$u(x, 0) = 0 \quad u(x, b) = 0 \quad 0 < x < a$$

$$u(0, y) = 0 \quad u(a, y) = f(y) \quad 0 \leq y \leq b$$

Where  $f$  is given function on  $0 \leq y \leq b$

- (3) (a) Define the Bessel equation  
(b) Assume that  $V$  is not an integer in the Bessel equation then show that

$$y = \sum_{n=0}^{\infty} c^n x^{m+r}$$

- (4) Show that  
(a)  ${}_2F_1(\alpha, \beta, \beta, x) = (1 - x)^{-\alpha}$   
(b)  ${}_2F_1(1; 1; 2; -x) = \log(1 + x)$