FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2023_1 POP EXAMINATION

Course Code: MTH 382
Course Title: Mathematical Methods IV
Credit Unit: 3
Time allowed: 3 HOURS
Instruction: Answer Question Number One and any Other Three Questions

1. (a) Define the following terms   (2 marks)
        (i) Hyper-geometric   (2 marks)
        (ii) Legendry function

    (b) Using the definition $\int_0^1 x^{m-1}(1-x)^{n-1}dx$, prove that

        $$B(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta cos^{2n-1}\theta dx$$ (11 marks)

    (c) Evaluate each of the following expressions.
        (i) $\frac{3\Gamma(6)}{\Gamma(4)}$ (5 marks)

        (ii) $\Gamma\left(\frac{3}{2}\right)$ (5 marks)

2. (a) Use the method of successive approximation to obtain the solution of the differential

    equation $\frac{dy}{dx} = 1 + xy$ up to the third approximation when $x_0 = 0$ and $y_0 = 0$ (6 marks)

    (b) $I = 2\int_0^\infty e^{-x^2}\,dx$ and $I = 2\int_0^\infty e^{-y^2}\,dy$. By finding an expression for I. Show that

        $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (9 marks)

3. (a) State the Bessel of the first kind of order v.   (2 marks)

    (b) Solve Bessel equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - v^2)y = 0$ (13 marks)

4. (a) State generating function for Legendre polynomials   (2 marks)

    (b) Using the Rodrigue's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$ to the Legendre
        polynomials
        (i) $P_4(x)$ (6 marks)
        (ii) $P_8(x)$ (7 marks)

5. (a) By using techniques involving the Beta function, find the exact value of

    (i) $\int_0^1 7x^5(1-x)^4 \, dx$ **(7 marks)**

    (ii) $\int_0^{\frac{\pi}{2}} \sin^7 \theta \, d\theta$ **(8 marks)**

6. (a) Show that the two definitions of gamma function are equivalent.   **(10 marks)**

    (b) State the relationship between gamma and beta functions.

        If $R(p) > 0$ and $R(q) > 0$ **(5 marks)**