NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
November 2018 Examinations

Course Code:	**MTH382**
Course Title:	**Mathematical Methods IV**
Credit Unit:	**3**
Time Allowed:	**3 Hours**
Total:	**70 Marks**
Instruction:	**Answer Question One (1) and Any Other 4 Questions**

1. (a) Use the series of the form $v(x) = 1 + c_1 \frac{1}{x} + c_2 \frac{1}{x^2} + c_3 \frac{1}{x^3} + \cdots$, to solve the differential equation $\frac{d^2v}{dx^2} + 2\frac{dv}{dx} + \frac{1}{4x^2} v(x) = 0$ **(10 marks)**

   (b) The gamma function is defined by the improper integral as $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$:

   (i) Prove that $\Gamma(x+1) = x\Gamma(x)$ **(6 marks)**

   (ii) Evaluate $\Gamma(x+1) = n!$ **(6 marks)**

2. (a) Use the generating function $f(t,x) = \frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x)\, t^n$ to obtain the recurrence relation $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$ **(6 marks)**

   (b) Use the recurrence $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$ to obtain the first four Legendre polynomials **(6 marks)**

3. (a) If p and q are positive integers, show that this relationship $B(p,q) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$ exist between the Beta and Gamma functions **(6 marks)**

   (b)Using the above relationship, evaluate (i) $\int_0^1 x^4 (1-\sqrt{x})^5 dx$ **(6 marks)**

4. Evaluate (a) $\int_0^\infty \sqrt[4]{x}\, e^{-\sqrt{x}} dx$ **(6 marks)**

   (b) $\int_0^\infty x^{n-1}\, e^{-h^2x^2} dx$ **(6 marks)**

5. (a) Use the Rodrigue's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2 - 1\right)^2$ to drive Legendre polynomial $P_4(x)$ **(6 marks)**

(b) Using the Rodrigue' formula, show that $f(x) = x^2$ can be written as a series of Legendre polynomials **(6 marks)**

6. (a) If $u(x, y) = e^x \cos y$ and $v = e^{-x} \sin y$, find the derivative of

(i) $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ **(2 marks)**

(ii) $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ **(2 marks)**

Hence, evaluate the Jacobian of $u(x, y)$ and $v(x, y)$ **(3 marks)**

(b) Solve the equation $\frac{\partial^2 u}{\partial x^2} = 12x^2(t+1)$ given that at $x = 0$, $u = \cos 2t$ and $\frac{\partial u}{\partial x} = \sin t$

(5 marks)