NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

**FACULTY OF SCIENCES**
**DEPARTMENT OF MATHEMATICS**
2023_1 POP EXAMINATION

**Course Code: MTH 401**
**Course Title: General Topology I**
**Credit Unit: 3**
**Time Allowed: 3 HOURS**
**Instruction: ATTEMPT NUMBER ONE (1) AND ANY OTHER THREE (3) QUESTIONS**

1. (a) Let (E, d) be a metric space and let A be a subset of E. When is point $p \in E$ called a boundary point of A? **(9 marks)**
   (b) Let E = R(the reals) with the usual metric, and let $Y = [0, 1] \cup (3, 4)$ as a subspace of E. Determine if each of the following sets is open or closed in Y.
   (i) $A = [0, 1]$ **(2 marks)**
   (ii) $B = (3, 4)$ **(2 marks)**
   (iii) $C = [0, \frac{1}{2})$. **(2 marks)**
   (c) Prove that $R^2$ is connected. **(10 marks)**

2. Prove that a metric space (E, d) is connected if and only if the only subsets of E which are both open and closed are E and $\emptyset$. **(15 marks)**

3. (a) Prove Pasting lemma of closed sets. **(8 marks)**
   (b) Let $f: R \rightarrow R$ be defined by $f(x) = x^2 + 1$, if $x \leq 0$ and $f(x) = \frac{1}{2}(x+2), if\ x \geq 0$. Show that f is continuous on R. **(7 marks)**

4. Let $f: R^2 \rightarrow R$ be defined by $f(x, y) = x^2 - 3xy + 5y^2 - 3x + 2y - 1$. Show that $f$ is continuous at $(-1, 2)$. **(15 marks)**

5. Let (E, d) be an arbitrary metric space and let $\{x_n\}$ be a Cauchy sequence in E. Prove that $\{x_n\}$ is bounded. **(15 marks)**

6. (a) Every subsequence of a convergent sequence converges, and it converges to the same limit as does the mother sequence. **(7 marks)**
   (b) Let $(E_1, d_1)$ and $(E_2, d_2)$ be two metric spaces and let $E = E_1 \times E_2$ denote their cartesian product, where E is endowed with its own metric. Define Euclidean metric on $E_1 \times E_2$. **(8 marks)**