NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES

April/May Examination 2019

Course Code:	**MTH401**
Course Title:	**General Topology 1**
Credit Unit:	**3**
Time allowed:	**3 HOURS**
Total:	**70 Marks**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Define a metric space (4marks)

   (b) Let $\mathbb{R}$ denote the set of real numbers and let $d: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be defined by $d(x, y) = |x - y|$ for all $x, y \in \mathbb{R}$. Show that d is a metric on $\mathbb{R}$. (6marks)

   (c) State Triangle and Hölder's inequalities (4marks)

   (d) State and prove Minkowski's inequality. (8marks)

2. (a) Define the following: (i) Open ball (ii) Closed ball (iii) Spheres. (5marks)

   (b) Let $E = \mathbb{R}$ be endowed with the Euclidean metric.

   $d_2(X, Y) = \sum_{i=1}^{k}\{(X_k - Y_k)^2\}^{1/2}$ for all $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.

   Describe the following sets(i)$B\big((0,0), 1\big)$(ii)$\overline{B}\ \big((0,0), 1\big)$(iii) $S((0,0), 1)$ where $(0,0) \in \mathbb{R}^2$(iv)$B_r(x_o)$ for arbitrary $x_0 \in \mathbb{R}^2$. (7marks)

3. (a) Define the closure of a set . (5marks)

   (b) Every singleton subset of any metric space is closed. Hence, every finite set is closed. (7marks)

4. (a) Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of points in a metric space (E, d). When is a point $x \in E$ said to be a limit point of the sequence $\{x_n\}$? (5marks)

   (b) Show that $\{x_n\}$converges to x in E, if and only if $\{d(x_n, x)\}$ converges to 0 in $\mathbb{R}$. (7marks)

5. (a) When is a sequence said to be a Cauchy in a metric space? (5marks)

   (b) Prove that every convergent sequence in a metric space is Cauchy. (7marks)

6. (a) Define a connected space (5marks)

   (b) Prove that the image of a connected space under a continuous map is connected. (7marks)