NATIONAL OPEN UNIVERSITY OF NIGERIA

Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES

DEPARTMENT OF MATHEMATICS

October Examination 2019

Course Code: **MTH 401**
Course Title: **General Topology I**
Credit Unit: **3**
Time Allowed: **3 Hours**
Instruction: **Answer Question Number One and Any Other Four Questions**

1. (a) Define the following terms:
   (i) condensing point **(3 marks)**
   (ii) accumulation point of F, where F is a subset of E of the metric space (E, d). **(4 marks)**
   (b) Let (E, d_E) be a metric space and let (Y, d_Y) be a subspace of X. Let A be subset of Y. Show that A is closed in Y if and only if there exists a set F which is closed in E such that $A = Y \cap F$. **(8 marks)**
   (c) Show that limits are unique in metric spaces (i.e. if $\{x_n\}$ converges to both x and x^t then $x = x^t$). **(7 marks)**

2. Let $f : R^2 \rightarrow R$ be defined by $f(x, y) = 3x^2 - 4xy + 7y^2 - 3x + 2y - 6$. Prove that f is continuous at $(-1, 6)$. **(12 marks)**

3. (a) Show that a subset F of a metric space (E, d) is closed in E if and only if its complement is open in E. **(7 marks)**
   (b) Show that every singleton subset of any metric space is closed. Hence, every finite set is closed. **(5 marks)**

4. (a) Let $(E_1\ d_1)$ and $(E_2\ d_2)$ be two metric spaces and let $E = E_1 \times E_2$ denote their cartesian product, where E is endowed with its own metric. Define Euclidean metric on $E_1 \times E_2$. **(7 marks)**
   (b) Show that every subsequence of a convergent sequence converges, and it converges to the same limit as does the mother sequence. **(5 marks)**

5. Prove that in any metric space (E, d), each open ball is an open set in E. **(12 marks)**

6. Show that a metric space (E, d) is connected if and only if the only subsets of E which are both open and closed are E and $\emptyset$. **(12 marks)**