NATIONAL OPEN UNIVERSITY OF NIGERIA

University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 Examinations

Course Code: **MTH411**
Course Title: **Measure Theory and Integration**
Credit Unit: **3**
Time Allowed: **3 Hours**
Instruction: **Attempt Number One (1) and any four (4) Questions**

1. (a) State Fatou's lemma **(3 marks)**

(b) Obtain m(F) given that F = [a, b], S = [a, b] and $C_sF = \emptyset$. **(3 marks)**

(c) Show that the measure of a bounded closed set F is non – negative. **(6 marks)**

(d) Let the bounded open set G be the union of finite or denumerable number of open sets G_k (that is, $G = \bigcup_k G_k$). Show that $m(G) \leq \sum_k m(G_k)$. **(10 marks)**

2. (a) State Holder's inequality. **(3 marks)**

(b) What is a point mass concentrated at x if (X, M) is a measurable space, $x \in X$ and fl ∈ M? **(3 marks)**

(c) Let (X, fl) have finite measure. Show that $L^p \subseteq L^R$ whenever $1 \leq r < p < \infty$. Moreover, the inclusion map from L^p to L^r is continuous. **(6 marks)**

3. (a) Define a q – algebra. **(6 marks)**

(b) Show that $m(G) \geq \sum_{k=1}^{n} M(I_k)$ if a finite number of pairwise disjoint open intervals $I_1, I_2, \ldots I_n$ are contained in an open interval G. **(6 marks)**

4. (a) State the four conditions f must satisfy on the measurable function f: A → [-∞, +∞]. **(6 marks)**

(b) Let (X, $\mathscr{M}$) be a measurable space, let A be a subset of X that belongs to $\mathscr{M}$, and let f and g be [-∞, +∞] - valued measurable functions on A. Show that $f \vee g$ and $f \wedge g$ are measurable. **(6 marks)**

5. (a) State (i) Monotone Convergence theorem. **(3 marks)**

(ii) Dominated Convergence theorem. **(4 marks)**

(b) Let X be an arbitrary set. State the properties of the collection Ω of subsets of X to be called an algebra. **(5 marks)**

6. Let (X, M) be a measurable space, and let fl be a finitely additive measure on (X, M). Show that fl is a measure if either
(i) $\lim_k$ fl(A_k) = fl($\bigcup_k A_k$) holds for each increasing sequence {A_k } of sets that belong to M. Or
(ii) $\lim_k$ fl(A_k) = 0 holds for each decreasing sequence {A_k} of sets that belong to M and satisfy $\bigcap_k A_k = \emptyset$. **(12 marks)**