NATIONAL OPEN UNIVERSITY OF NIGERIA
University Village, Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2023_1 POP EXAMINATION

Course Code: **MTH 411**
Course Title: **Measure Theory and Integration**
Credit Unit: **3**
Time Allowed: **3 HOURS**
Instruction: **ATTEMPT NUMBER ONE(1) AND ANY OTHER THREE(3) QUESTIONS**

1. (a) Explain the following terms:
(i) The measure of a bounded open set. **(4 marks)**
(ii) The measure of a non – empty bounded closed set F. **(4 marks)**
(iii) Minkowski inequality. **(4 marks)**
(b) Given $f_n: X \to \mathbb{R}$ is a sequence that converge pointwise to f. There exist an integrable function g such that $|f_n| \leq g$. Prove that $f_n$and f are also integrable and $\lim_{n\to\infty} \sup \int |f_n - f| = 0$. **(13 marks)**

2. (a) Let (X, M) be a measurable space. Show that counting measure on $(X, M)$is a measurable space. **(6 marks)**
(b) Distinguish between measurable function and Borel function with four examples. **(9 marks)**

3. (a)) Let (X, M, fl) be a measure space, and let f and g be extended real-valued functions on X that are equal almost everywhere. If fl is complete and if f is measurable, explain that g is measurable. **(7 marks)**
(b)) Let G_1 , G_2 be open sets such that $G_1 \subseteq G_2$, prove that $m(G_1) \leq m(G_2)$. **(8 marks)**

4. (a) What is a simple function? **(3 marks)**
(b) Let $\Sigma_{n=1} \int_x | n | d$ fl < ∞. Prove that $\Sigma_{n=1}$ $_n$(x) converges to f(x) on X. **(12 marks)**

5. (a) State Beppo Levi's theorem. **(4 marks)**
(b) Let $A \subseteq B$. Prove that fl (A) ≤ fl (B) satisfies fl(A) < +∞, then fl(B – A) = fl(B) – fl(A). **(11 marks)**

6. (a) Using monotone convergence, prove that $\int \sum_{n=1}^{\infty} f_n = \sum_{n=1}^{\infty} \int f_n$ **(7 marks)**
(b) Let fl be Lebesque measure on $\mathbb{R}$ and f is a function defined on $\mathbb{R}$ as follows:
f(x) = {$4\ if -3 < x < 3; 5\ if\ 3 \leq x < 7; 8\ if\ 7 \leq x < 9; 1\ if -7 < x \leq -3;$
2 if $-9 < x \leq -7$; 0 otherwise. Find $\int f(r)$fl(dr). **(8 marks)**