NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
April Examination 2019

Course Code:	**MTH411**
Course Title:	**Measure Theory and Integration**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Total:	**70 Marks**
Instruction:	**ATTEMPT NUMBER ONE AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Let G be a bounded non empty open set. Define the measure of G **(2marks)**

(b)Prove that If a finite number of pairwise disjoint open intervals $I_1, I_2, I_3, \ldots I_n$ are contained in an open interval G, then $m(G) \geq \sum_{k=1}^{n} m(I_k)$. **(9marks)**

(c) i. Prove that : The measure of a bounded closed set F is non – negative. **(5marks)**

ii. Let F be a closed set and let G be a bounded open set. If F ⊆ G, Prove that m(F) ≤ m(G). **(6marks)**

2. (a) (i) Define the outer measure of a bounded set. **(2marks)**

(ii) Define the inner measure of a bounded set **(2marks)**

(b) (i) Prove that for any bounded set E, $m_*(E) \leq m^*(E)$. **(3marks)**

(ii) If a bounded set E is the union of a finite or denumerable number of sets E_k, **(5marks)**

E = $\bigcup_k E_k$. show that $m^*(E) \leq \sum_k m^*(E_k)$.

3. (a) (i) What is a measurable function? **(3marks)**

(ii) Define a simple function **(1mark)**

(b) Let (X, M, μ) be a measure space. If $\{A_k\}$ is an arbitrary sequence of sets that belong to M , show that

$$\mu\left(\bigcup_{k=1}^{\infty} A_k\right) \leq \sum_{k=1}^{\infty} \mu(A_k)$$ **(8marks)**

4. a. (i) Define a measurable space **(2marks)**

(ii) Define Borel $\sigma -$ algebra on $\mathbb{R}^n$ **(2marks)**

b. Let X be any non – empty set. Show that the intersection of an arbitrary non – empty collection of $\sigma -$ algebras on X is a $\sigma -$ algebra on X. **(8marks)**

5. a Let (X, $\mathcal{M}, \mu$) be a measure space, and let f and g be extended real-valued functions on X that are equal almost everywhere. If μ is complete and if f is measurable, show that g is measurable. **(5marks)**

b. (i) Define algebra in measure theory **(4marks)**

(ii) What is sigma algebra? **(3marks)**

6 a. Let (X, $\mathcal{M}$) be a measurable space, let A be a subset of X that belongs to $\mathcal{M}$, and let $\{f_n\}$ be a sequence of [-∞, +∞] - valued measurable functions on A. Show that

(i) The functions $sup_n f_n$ and $inf_n f_n$ are measurable **(3marks)**

(ii) The functions $limsup_n f_n$ and $liminf_n f_n$ are measurable **(3marks)**

(III) The function $lim_n f_n$ is measurable. **(3marks)**

b. (i) What is a measure on a measurable space? **(1marks)**

(ii) What is a finitely additive function? **(2marks)**