NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
November Examination 2018

Course Code:	**MTH411**
Course Title:	**Measure Theory**
Credit Unit:	**3**
Time Allowed:	**3 HOURS**
Total:	**70 Marks**
Instruction:	**ATTEMPT NUMBER ONE (1) AND ANY OTHER FOUR (4) QUESTIONS**

1. (a) Let A and B be bounded sets such that A ≤ B. Show that:
   - (i) M_x (A) ≤M_x (B) (5marks)
   - (ii) M^x (A) ≤M^x (B) (5marks)

   (b) Let a bounded set E be the union of a finite number of sets E_k, prove that M^x (E) $\leq \sum_k M^x(\mathrm{Ek})$ (6marks)

   (c) Suppose a bounded set E is the union of a denumerable number of pairwise disjoint sets E_k. Show that M_x (E) $\geq \sum_k Mx(\mathrm{Ek})$ (6marks)

2. (a) Define a simple function and give an example (3marks)

   (b) Define a characteristic function (2marks)

   (c) Let (X, M) be a measurable space and A be a subset of X belonging to M. Let f and g be [-∞, +∞]. Show that f∨g and f∧g are measurable. (7marks)

3. (a) Define the measure of a bounded set (3marks)

   (b) Prove that m(G_1) ≤ m(G_2) given that G_1, G_2 are open sets. (4marks)

   (c) Let the bounded open set G be the union of finite number of open sets G_k. Show that m(G) $\leq \sum_k \mathrm{m}(\mathrm{Gk})$. (5marks)

4. a. Show that μ is a measure if $\lim_k \mu(Ak)$ = μ $(\bigcup_k Ak)$, where A_k is an increasing sequence of sets belonging to M (4marks)

   b. Prove that $\lim_k \mu(Ak)$ = 0, where A_k is a decreasing sequence of sets belonging to M (4marks)

   c. When is a function said to be measurable? Give an example of a function that is Borel measurable? (4marks)

5. a What do you understand by the outer measure of a bounded set E? (3marks)

   b. Define the least upper bound of the measures of all closed sets. (3marks)

   c. Discuss the statement “a set is measurable” (6marks)

6 a. Define a set function (3marks)

   b. When is a measurable space said to be countable additive? And when is it finitely additive? (6marks)

   c. Define a measurable space (3marks)