NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Expressway. Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code: MTH 411
Course Title: Measure Theory and Integration
Credit Unit: 3
Time Allowed: 3 Hours
Instruction: Answer Question Number One and Any other Four Questions

1. (a) Define outer and inner measures of bounded sets. **(8 marks)**

   (b) Show that for any bounded set E, $m_*(E) \leq m^*(E)$. **(7 marks)**

   (c) Let G_1, G_2 be open sets such that $G_1 \subseteq G_2$. Show that $m(G_1) \leq m(G_2)$. **(7 marks)**

2. (a) Show that $m(G) \geq \sum_{k=1}^{n} M(I_k)$ if a finite number of pairwise disjoint open intervals $I_1, I_2, \ldots I_n$ are contained in an open interval G. **(6 marks)**

   (b) Suppose that $f1$ is a Lebesque measure and that f is defined as follows:
   f(x) = {$3\ if - 2 < x < 2; 4\ if\ 2 \leq x < 6; 6\ if\ 6 \leq x < 11; 1\ if - 6 < x \leq -2$; 2 if -11< x ≤ -6; 0 otherwise. Find $\int f(r) f1(dr)$ **(6 marks)**

3. (a) State Fatou's lemma. **(3 marks)**

   (b) Let the bounded open set G be the union of finite or denumerable number of open sets G_k (that is, $G = \bigcup_k G_k$). Show that $m(G) \leq \sum_K m(G_k)$. **(9 marks)**

4. (a) Define the measure of a non – empty bounded closed set F. **(4 marks)**

   (b) Obtain m(F) given that F = [a, b], S = [a, b] and $C_sF = \emptyset$. **(4 marks)**

   (c) Show that the measure of a bounded closed set F is non – negative. **(4 marks)**

5. (a) State Minkowski's Inequality **(3 marks)**

(b) Let $f_n: X \to [0, \infty]$ be non – negative measurable functions. Show that

$\int \sum_{n=1}^{\infty} f_n = \sum_{n=1}^{\infty} \int f_n$.(Beppo – Levi) **(4 marks)**

(c) Let (X, M, fl) be a measure space and let A and B be subsets of X that belong to M and satisfy A $\subseteq B$. Show that fl (A) $\leq$ fl (B). If in addition A satisfies fl(A) < +∞, then fl(B – A) = fl(B) – fl(A). **(5 marks)**

6. (a) Define a q – algebra **(4 marks)**

(b) Give four (4) examples of a q – algebra **(8 marks)**