NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021_2 EXAMINATION

Course Code: **MTH421**
Course Title: **Ordinary Differential Equations**
Credit Unit: **3**
Time Allowed: **3 Hours**
Total: **70 Marks**
Instruction: **Answer Question Number One and Any Other Four Questions**

1. (a) Define the following .

i. Connected Set . **(2 marks)**

ii. Open Set **(2 marks)**

iii. Boundary Point **(2 marks)**

iv Convergence of a sequence of real numbers $\{x_n\}$ **(2 marks)**

v. Solution to an ODE **(2 marks)**

(b) . (i) Find the general solution to the ODE

$$3y'' + 2y' - 2y = 0.$$

(4 marks)

(ii) Solve the 1st order ODE

$$\frac{dy}{dx} + y\tan x = \sec x.$$

(4 marks)

(c) Obtain the general solution to the equation:

$$(x^2 + 1)\frac{dy}{dx} + (y^2 + 1) = 0.$$

(4 marks)

2. (a) Find the particular solution to the ordinary differential equation:

$$x\frac{dy}{dx} = y + x, \;\; y(0) = 0.$$

(4 marks)

(b) Solve the ODE: $y' = -7y$ **(4 marks)**

(c) Verify if the ODE $(1 + 2xy^3)dx + 3x^2y^2dy = 0$ is exact. **(4 marks)**

3. (a) Solve the ODE:

$$\frac{2x}{y}y' = \frac{y^2}{x^2}.$$

(6 marks)

(b) Solve the exact ODE:

$$\frac{1}{3}x^3y^3dx + \frac{1}{4}x^4y^2dy = 0.$$

. **(6 marks)**

4. (a) Show that the general solution to the ODE:

$$-\frac{1}{x}\cos xy\,dx - \frac{1}{y}\cos xy\,dy = 0.$$

satisfies the equation:

$$f(x) = c + \frac{\sin xy}{xy}, \quad c\ constant.$$

. **(6 marks)**

(b) Solve the ODE:

$$\frac{1}{2}x^2y^2dx + \frac{1}{3}x^3ydy = 0.$$

(6 marks)

5. (a) Define Exact ODE **(2 marks)**

(b) Solve the IVP: $y' + y\tan x = \sin 2x\,,\ \ y(0) = 1.$ **(5 marks)**

(c) Solve the IVP: $y' - y = e^{2x}$. **(5 marks)**

6. (b) Find a general solution to the following system of ODEs:

$$y_1' = 3y_2,$$
$$y_2' = 12y_1.$$

(4 marks)

(a) Solve the following system:

$$y_1' = 9y_1 + 13.5y_2,$$
$$y_2' = 1.5y_1 + 9y_2.$$

(8 marks)