NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021 Examination 1245

Course Code: MTH421
Course Title: Ordinary Differential Equations
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question Number One and Any Other Four Questions

1. (a) Define the following .
    i. Initial value problem (IVP).. .   **(3 marks)**
    ii. Domain   **(1 mark)**
    iii. Closed Domain   **(1 mark)**

(b) . Find the total differentials of the following functions:
    (i) $u(x,y) = x^2 \cos x + 3x^2 y^3$.   **(2 marks)**
    (ii) $p(x,y) = 2x^3 y^2 + xy^3$.   **(2 marks)**
    (iii) $m(x,y) = 3x^2 y^5 + 7x^3 y^2$.   **(2 marks)**

(c)   i. Define five types of critical points   **(5 marks)**
    ii. Solve the IVP using integrating factor
      $y' = 3y, \ y(0) = 5.7$.   **(6 marks)**

2.   (a) Solve the ODE $y' = 1 + y^2$.   **(4 marks)**
    (b) Find the particular solution to the IVP: $y' = ky, \ y(0) = y_0$. **(4 marks)**
    (c) Differentiate:
      i. $n(x,y) = 7x^4 y - 5xy$.   **(2 marks)**
      ii. $q(x,y) = \sqrt{2}x^2 + y$.   **(2 marks)**

3.   (a) Solve the ODE: $\cos(x+y)\,dx + (3y^2 + 2y + \cos(x+y))dy = 0$
                                   (6 marks)

  (b) Find the general solution to the ODE: $\left(x^2 y + \frac{1}{3}y^3\right)dx + \left(\frac{1}{3}x^3 + xy^2\right)dy = 0$.
    .   **(6 marks)**

4. (a) Obtain the solution to the following equation: $\frac{1}{8}xy^8 dx + \frac{1}{2}x^2y^7 dy = 0$.

   . **(6 marks)**

   (b) Find the particular solution to the initial value problem:
   $y'' + y = 0$, $y(0) = 3$, $y'(0) = 0.5$. **(6 marks)**

5. (a) Solve the 2nd-order ODE: $y'' + y' - 2y = 0$, $y(0) = 4$, $y'(0) = 5$.

   (b) Obtain the particular solution to:
   $y'' + y' + 0.25y = 0$, $y(0) = 3.0$, $y'(0) = 3.5$. **(6 marks)**

6. (a) Find the general solution to: $y'' + 0.4y' + 0.4y = 0$. **(5 marks)**

   (b) Solve the following system of ODEs:

   $y_1' = 8y_1 - y_2$,
   $y_2' = y_1 + 10y_2$.

   (7 marks)