NATIONAL OPEN UNIVERSITY OF NIGERIA
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FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code: MTH421
Course Title: Ordinary Differential Equation
Credit Unit: 3
Time Allowed: 3 Hours
Total: 70 Marks
Instruction: Answer Question Number One and Any Other Four Questions

1 (a) (i) State the Lyapunov theorem of a system **(3 marks)**

(ii) Define positive definite and positive semi definite of a system. **(3 marks)**

(b). Show that the solution $e^{x}, e^{-x} and\ e^{2x}$ of $y''' - 2y'' - y' + 2y = 0$

are linearly independent. **(6marks)**

(c) Consider the system $\dot{X}_1 = x_1^3 - 2x_1x_2^2,$
$\dot{X}_2 = x_1^2x_2 - x_2^3$

with the function $V(x_1, x_2)$ defined by $V(x_1, x_2) = x_1^2 + x_1^2x_2^2 + x_2^4$
prove that the system is asymptotically stable in the sense of Lyapunov **(10marks)**

2. (a). Define and describe stability at a critical point. **(3marks)**

(b) Show that a necessary and sufficient condition that f be solution of the nth order ODE

$L_ny = 0$ is the solution of the $(n-1)$ first order linear differential equation $P[y, g(x)] = c$, where $P(u,v)$ is the bilinear concomitant associate with L_n, the function g is a nontrivial solution of the adjoint equation $\overline{L_n}\ y = 0$, and c is arbitrary constant. **(5marks)**

(c) Consider the non-linear system
$$\dot{x} = x + 4y - x^2$$
$$\dot{y} = 6x - y + 2xy$$

Determine the type and stability at the critical point $(0,0)$ of the nonlinear autonomous system. **(4marks)**

3. (a) Define the terms Lipschitz continuity and Lipschitz constant of a function in a region.

(4marks)

(b). Find Picards first three approximations y_1, y_2, y_3 for the initial value problem

$y' = x^2 + y, \quad y(0) = 1, \qquad D: |x| \le 1, \qquad |y-1| \le 1$. Determine the largest interval on which the theorem guaranteed the existence and uniqueness of a solution. **(4marks)**

(c) Find nontrivial solutions of the Sturm-Liouville problem

$\frac{d^2y}{dx^2} + y = 0, \quad y(0) = 0 \ , \ y'(0) = 0$. **(4marks)**

4.(a) Solve the differential equations

$(i)\ (1+x)\,y\,dx + (1-y)\,x\,dy = 0 \qquad (ii)\ \frac{dy}{dx} = \frac{1+y}{1+x}$ **(4marks)**

(b) Convert the differential equation $x^2y'' - xy' + y = \log x$ into a differential equation with constant coefficients. **(4marks)**

(c) Let I_n be the adjoint operator on L_n. Show that L_n is the adjoint operator of $\overline{L_n}$. **(4marks)**

5. (a) Discuss solution of the homogeneous linear system. **(4marks)**

(b) Solve the system $\dot{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ \sin t \end{pmatrix}, \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ **(4marks)**

(c) Solve the differential equation $\dddot{X} - 2\ddot{X} - \dot{X} + 2X = 0$,

subject to $x(0) = 0, \dot{x}(0) = 0, \ddot{x}(0) = 1$ **(4marks)**

6. (a) Describe the method of undetermined coefficients. **(3marks)**

(b) Find the general solution of $y^{iv} - 5y''' + 6y'' + 4y' - 8y = 0$ **(4marks)**

(c) Using the method of undetermined coefficients solve the differential equation

$y'' - 2y' - 3y = 2e^x - 10\sin x, \quad y(0) = 2, \ y'(0) = 4.72$ **(5marks)**