NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikiwe Expressway, Jabi, Abuja.

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
2021 EXAMINATION

Course Code: **MTH423**
Course Title: **Integral Equations**
Credit Unit: **3**
Time Allowed: **3 Hours**
Total: **70 Marks**
Instruction: **Answer Question Number One and Any Other Four Questions**

1. (a) i. Define the three types of Volterra integral equations **(3 marks)**
ii. State three properties of Volterra integral equations. **(3 marks)**

(b) . i. What is a degenerate Kernel? **(2 marks)**
ii. When is the set of orthogonal system $\{\phi_n\}$ said to be complete? **(4 marks)**
iii. When is a Kernel said to be positive? **(3 marks)**
iv. State the Hilbert-Schmidt Theorem. **(3 marks)**

(c) Find the solution to the integral equation:

$$\frac{a}{a^2+x^2} = \int_0^\infty \cos \omega x \, \phi(\omega) d\omega , \quad a > 0.$$

(4 marks)

2. (a) Solve the integral equation:

$$\int_0^x \sin x \,(x-y) dy = 1 - \cos \beta x.$$

(6marks)

(b) Form the integral equation corresponding to the IVP:

$$y'' + 2xy' + y = 0, \;\; y(0) = 1, \qquad y'(0) = 0.$$

. **(6 marks)**

3. (a) Solve the integral equation:

$$\phi(x) = \lambda \int_0^\infty \cos \omega x \, \phi(\omega) d\omega \,, \quad \phi(x) \; is \; an \; even \; function \; of \; x.$$

(6 marks)

(b) Obtain the solution to the integral equation:

$$\phi(x) = f(x) + \lambda \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \int_0^\infty \cos xy \, \phi(y) dy.$$

. **(6 marks)**

4. (a) Using $\phi_0(x) = x$ as a first approximation, solve the integral equation:

$$\phi(x) = x + \lambda \int_0^\infty \phi(s) ds.$$

(b) Solve the integral equation:

$$\phi(x) = x^2 + \lambda \int_0^\infty x^3 s^2 \phi(s) ds.$$

5. (a) Solve the integral equation:

$$\phi(x) = x^5 + \int_0^\infty x s^2 \phi(s) ds.$$

(b) Find the solution of the integral equation:

$$u(x) = e^x + \frac{1}{e} \int_0^\infty u(y) dy$$

by the method of successive approximations.

6. (a) Find the eigenvalues and the corresponding eigenfunctions of the following homogeneous Fredholm integral equation

$$y(x) = \lambda \int_0^1 (\sin \pi x \cos \pi t) y(t) dt.$$

(6 marks)

(b) Show that the following homogeneous Fredholm integral equation has no eiigenvalues and no eigenfunctions:

$$y(x) = \lambda \int_0^1 (3x - 2) t \, y(t) dt.$$

(6 marks)