**NATIONAL OPEN UNIVERSITY OF NIGERIA**
**Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja**

**FACULTY OF SCIENCES**
**November, 2018 Examinations**

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| **Course Code:** | **MTH423** |
| **Course Title:** | **Integral Equations** |
| **Credit Unit:** | **3** |
| **Time Allowed:** | **3 Hours** |
| **Total:** | **70 Marks** |
| **Instruction:** | **Answer Question One (1) and Any other Four (4) Questions** |

1.(a) Define the following equations:

(i) An integral equation **(2 marks)**

(ii) Fredholm integral equation **(2 marks)**

(iii) Volteral integral equation **(2 marks)**

(b) Classify the following equations

(i) $u(x) = x - \frac{1}{6}x^3 - \int_0^x (x-t)u(t)dt = 0$ **(2 marks)**

(ii) $u(x) = \frac{1}{2} + x - \int_0^1 (x-t)\ u^2(t)dt = 0$ **(2 marks)**

(iii) $u'(x) = 1 - \frac{1}{3}x^3 + x - \int_0^x t\,u(t)dt = 0$ **(2 marks)**

(iv) $u(x) = \int_0^1 (x-t)^2\ u(t)dt = 0$ **(2 marks)**

(c) (i) Show that $u(x) = e^x$ is a solution of the equation

$$u(x) = 1 + \int_0^x u(t)dt$$ **(3 marks)**

(ii) Show that $u(x) = x$ is a solution of the integral equation

$$u(x) = \frac{5}{6}x - \frac{1}{9} + \frac{1}{3}\int_0^1 (x+t)u(t)dt$$ **(5 marks)**

2. (a) Convert the Volterra integral equation to an initial problem

$$u(x) = x + \int_0^x (t-x)u(t)dt$$ **(5 marks)**

(b) Solve the Fredholm integral equation

$$u(x) = \frac{9}{10}x^2 + \int_0^1 \frac{1}{2}x^2t^2u(t)dt$$ **(7 marks)**

3. (a) Use the transformation formula $\int_0^x \int_0^{x_1} \int_0^{x_2} \cdots \int_0^{x_{n-1}} f(x_n) dx_n \cdots dx_1 = \frac{1}{(n-1)!} \int_0^x (x-t)^{n-1} f(t) dt$,

convert the quadruple integral to a single integral.

$\int_0^x \int_0^x \int_0^x \int_0^x u(t) dtdtdtdt$ **(3 marks)**

(b) Convert the following initial value problem to an equivalent Volteral integral equation

$y''' - 3y'' - 6y' + 5y = 0$ $y(0) = y'(0) = y''(0) = 1$ **(9 marks)**

4. (a) Define degenerate kenels for Fredholm equations **( 2 marks)**

(b) Solve the Volterra integral equation $u(x) = x^2 + \frac{1}{12}x^4 + \int_0^x (t-x)u(t)dt$ by converting it to equivalent initial value problem **(10 marks)**

5. ( a) Define Sturm-Liouville problem **(2 marks)**

(b) Given the equation $\frac{\mathrm{d}^2\mathrm{y}}{dx^2} + 4y = f(x)$with $y(0) = 0, y\left(\frac{\pi}{4}\right) = 0$ . Determine the integral formulation of the problem **(10 marks)**

subject to boundary conditions $y(0) = 1, y(\pi) = \pi - 1$

6. (a) Show that u(x) = sinhx is a solution of the Volterra integral equation

$u(x) = x + \int_0^x (\mathrm{x} - \mathrm{t})\ \mathrm{u(t)}\, dt,$ **(6 marks)**

(b) Solve the Volterra integral equation $\int_0^x Sin\, \alpha(\mathrm{x}-\mathrm{y})\, \varphi(\mathrm{y})\, dy = 1 - \cos \beta x$ **(6 marks)**