NATIONAL OPEN UNIVERSITY OF NIGERIA
Plot 91, Cadastral Zone, Nnamdi Azikwe Express Way, Jabi-Abuja

FACULTY OF SCIENCES
DEPARTMENT OF MATHEMATICS
October Examination 2019

Course Code: **MTH423**
Course Title: **Integral Equations**
Credit Unit: **3**
Time Allowed: **3 Hours**
Total: **70 Marks**
Instruction: **Answer Question Number One and Any Other Four Questions**

1.(a) Define the following equations:

(i) An integral equation **(2 marks)**

(ii) Fredholm integral equation **(2 marks)**

(iii) Volteral integral equation **(2 marks)**

(b) Classify the following equations

(i) $u(x) = x - \frac{1}{6}x^3 - \int_0^x (x-t)u(t)dt = 0$ **(2 marks)**

(ii) $u(x) = \frac{1}{2} + x - \int_0^1 (x-t)\ u^2(t)dt = 0$ **(2 marks)**

(iii) $u'(x) = 1 - \frac{1}{3}x^3 + x - \int_0^x t\, u(t)dt = 0$ **(2 marks)**

(iv) $u(x) = \int_0^1 (x-t)^2\, u(t)dt = 0$ **(2 marks)**

(c) (i) Show that $u(x) = e^x$ is a solution of the equation

$$u(x) = 1 + \int_0^x u(t)dt$$ **(3 marks)**

(ii) Show that $u(x) = x$ is a solution of the integral equation

$$u(x) = \frac{5}{6}x - \frac{1}{9} + \frac{1}{3}\int_0^1 (x+t)u(t)dt$$ **(5 marks)**

2. (a) Convert the Volterra integral equation to an initial problem

$$u(x) = x + \int_0^x (t-x)u(t)dt$$ **(5 marks)**

(b) Solve the Fredholm integral equation

$$u(x) = \frac{9}{10}x^2 + \int_0^1 \frac{1}{2}x^2t^2u(t)dt$$ **(7 marks)**

3. (a) Use the transformation formula $\int_0^x \int_0^{x_1} \int_0^{x_2} \cdots \int_0^{x_{n-1}} f(x_n)dx_n \cdots dx_1 = \frac{1}{(n-1)!}\int_0^x (x-t)^{n-1} f(t)dt$,

convert the quadruple integral to a single integral.

$\int_0^x \int_0^x \int_0^x \int_0^x u(t)dtdtdtdt$ **(3 marks)**

(b) Convert the following initial value problem to an equivalent Volteral integral equation

$y''' - 3y'' - 6y' + 5y = 0$ $y(0) = y'(0) = y''(0) = 1$ **(9 marks)**

4. (a) Define degenerate kenels for Fredholm equations **(2 marks)**

(b) Solve the Volterra integral equation $u(x) = x^2 + \frac{1}{12}x^4 + \int_0^x (t-x)u(t)dt$ by converting it to equivalent initial value problem **(10 marks)**

5. (a) Define Sturm-Liouville problem **(2 marks)**

(b) Given the equation $\frac{d^2y}{dx^2} + 4y = f(x)$with $y(0) = 0, y\left(\frac{\pi}{4}\right) = 0$. Determine the integral formulation of the problem **(10 marks)**

subject to boundary conditions $y(0) = 1, y(\pi) = \pi - 1$

6. (a) Show that u(x) = sinhx is a solution of the Volterra integral equation

$u(x) = x + \int_0^x (x-t)\ u(t)\,dt,$ **(6 marks)**

(b) Solve the Volterra integral equation $\int_0^x Sin\,\alpha(x-y)\,\varphi(y)\,dy = 1 - \cos\beta x$ **(6 marks)**